

# Initial–Final-State Interference in the Z line-shape

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## Abstract

The uncertainty in the determination of the Z line-shape parameters coming from the precision of the calculation of the Initial-State Radiation and Initial–Final-State Interference is  $2 \times 10^{-4}$  for the total cross section  $\sigma_{had}^0$  at the Z peak, 0.15 MeV for the Z mass  $M_Z$ , and 0.1 MeV for the Z width  $\Gamma_Z$ . Corrections to Initial–Final-State Interference beyond  $\mathcal{O}(\alpha^1)$  are discussed.

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High-precision and high-statistics LEP measurements performed at c.m.s. energies close to the Z mass provide the most stringent tests of the Standard Model. In the near future LEP experiments will publish results of the final analysis of the Z line shape. The expected precision of the combined results of the four LEP experiments is about 2 MeV on the Z mass  $M_Z$  and width  $\Gamma_Z$ , and 0.1% on the  $\sigma_{had}^0$  measurement [1]. The uncertainty in  $\sigma_{had}^0$  includes an improved precision of the theoretical calculations of the small-angle Bhabha scattering of 0.061% [2, 3].

Large theoretical corrections are needed to extract the line-shape parameters  $M_Z$ ,  $\Gamma_Z$  and  $\sigma_{had}^0$  from the experimentally measured cross-sections. The largest correction, about 30% on the Z peak, comes from Initial-State Radiation (ISR), and it was discussed in Ref. [4]. In view of the high precision of the ISR calculation [4] and of the measurements, the precision of the calculation of Initial–Final-State Interference (IFI) becomes important as well. Results of independent calculations of this contribution in  $\mathcal{O}(\alpha^1)$  are compared here. The new results at higher orders are given.

Two line-shape fitting programs are used by LEP experiments: ZFITTER [5] and MIZA [6]. ZFITTER was cross-checked with TOPAZ0 [7]. The influence of the precision of the IFI calculation on the precision of the fitted Z line-shape parameters is studied here in the c.m.s. energy region between 88 and 94 GeV. This study was done with the fitting program MIZA, as used by the ALEPH Collaboration.

It is important to stress that the theoretical answer for pure  $\mathcal{O}(\alpha^1)$  IFI is known *unambiguously*, since the exact formula for the  $\gamma$ -Z box was published in Ref. [8] and the exact single real bremsstrahlung matrix element was given in Ref. [9]. Since then, the only problem left is purely technical, i.e. the phase-space integration. Contrary to other calculations<sup>1</sup>, the pure  $\mathcal{O}(\alpha^1)$  subgenerator of KORALZ<sup>2</sup> features the *exact* phase-space integration. Furthermore, in checking the exact analytical calculations in Ref. [11] for the IFI, results from  $\mathcal{O}(\alpha^1)$  KORALZ were cross-checked to the precision level of  $10^{-4}$ . In refs. [12, 13] (mainly, but not exclusively, devoted to forward–backward asymmetry), some important details on the technical side of these high-precision tests are also given. This  $\mathcal{O}(\alpha^1)$  subgenerator of KORALZ is at present the best available benchmark for the calculation of the  $\mathcal{O}(\alpha^1)$  IFI, for *any* experimental cuts.

Measured experimental cross-sections are extrapolated to full angular acceptance and to low  $s'$  before the Z line-shape fit is performed. The published cross sections for the ALEPH Collaboration [14] correspond to the  $s'/s$  cut of  $(0.1)^2$  for hadronic final states and to  $(2m_\tau)^2/M_Z^2$  for leptonic final states. Figure 1 shows good agreement between different  $\mathcal{O}(\alpha^1)$  calculations of IFI for the leptonic Z decays in MIZA [6], using formulas from Ref. [11], ZFITTER [5], as described in [15], and, what is very important, also with the  $\mathcal{O}(\alpha^1)$  of KORALZ [16].

The IFI correction for hadronic Z decays is about ten times smaller, owing to partial cancellation of contributions from different quark flavours. Relative IFI correction to both

<sup>1</sup> The IFI calculation for an acollinearity cut-off in ZFITTER is based on certain approximations, albeit justifiable near Z.

<sup>2</sup> The pure  $\mathcal{O}(\alpha^1)$  subgenerator of KORALZ is based on the MC of Ref. [10]; however, this was never tested below a 2% precision level, and it contained the approximate  $\gamma$ -Z box.

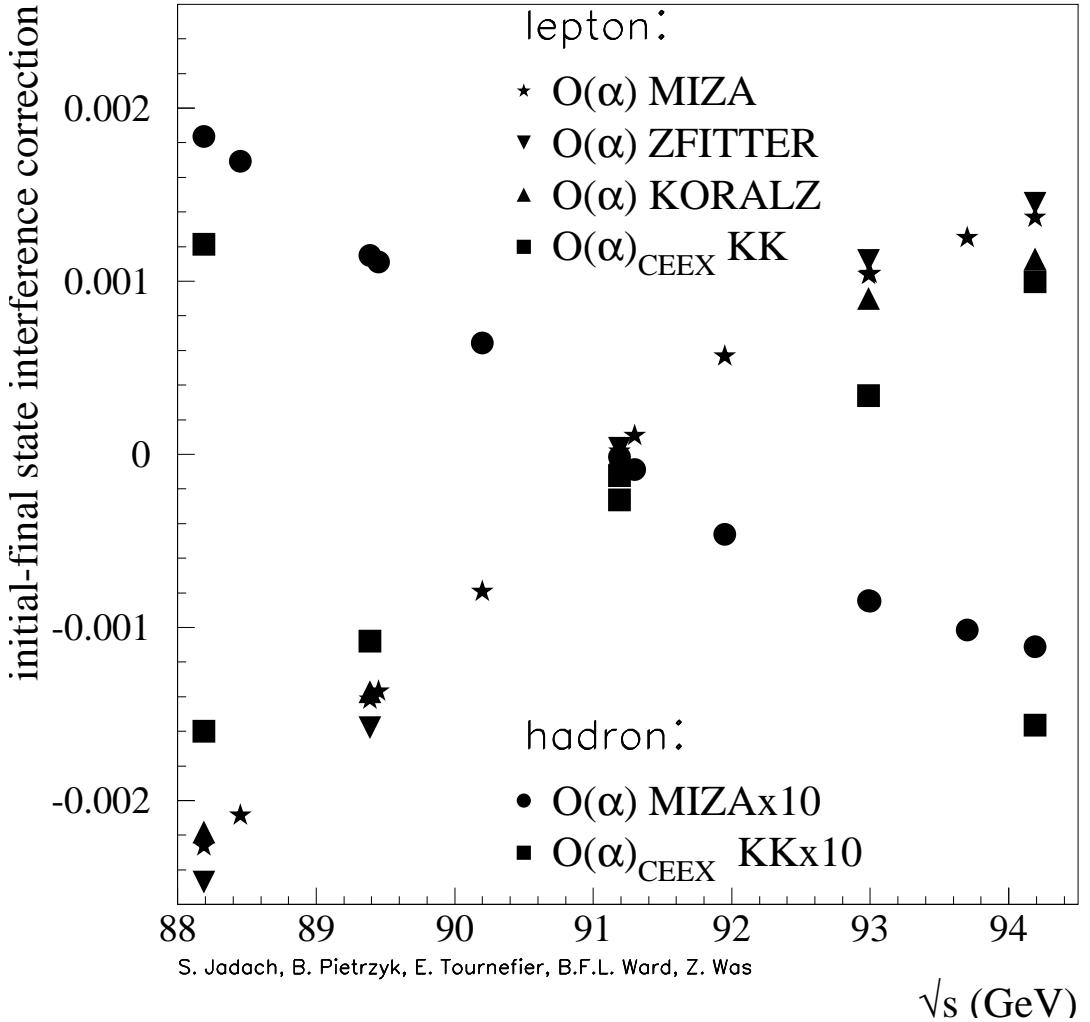


Figure 1: Comparison of different implementations of Initial–Final-State Interference. Numbers are relative to the cross-sections.

leptonic and hadronic cross section is very well approximated by a straight line, with zero at peak position. It influences, therefore, only the fitted value of the Z mass  $M_Z$  [17]. As seen in Table 1 the absolute contributions of IFI to both hadronic and leptonic cross sections, and therefore to the fitted  $M_Z$  are simmilar. The slope of the IFI correction in Fig. 1 in the hadronic and leptonic cases is opposite and the combined effect of the respective contributions to  $M_Z$  is negligible.

This conclusion is so far limited to  $\mathcal{O}(\alpha^1)$  and the important question is: Can higher orders change the IFI-integrated cross section significantly? Let us investigate the basic

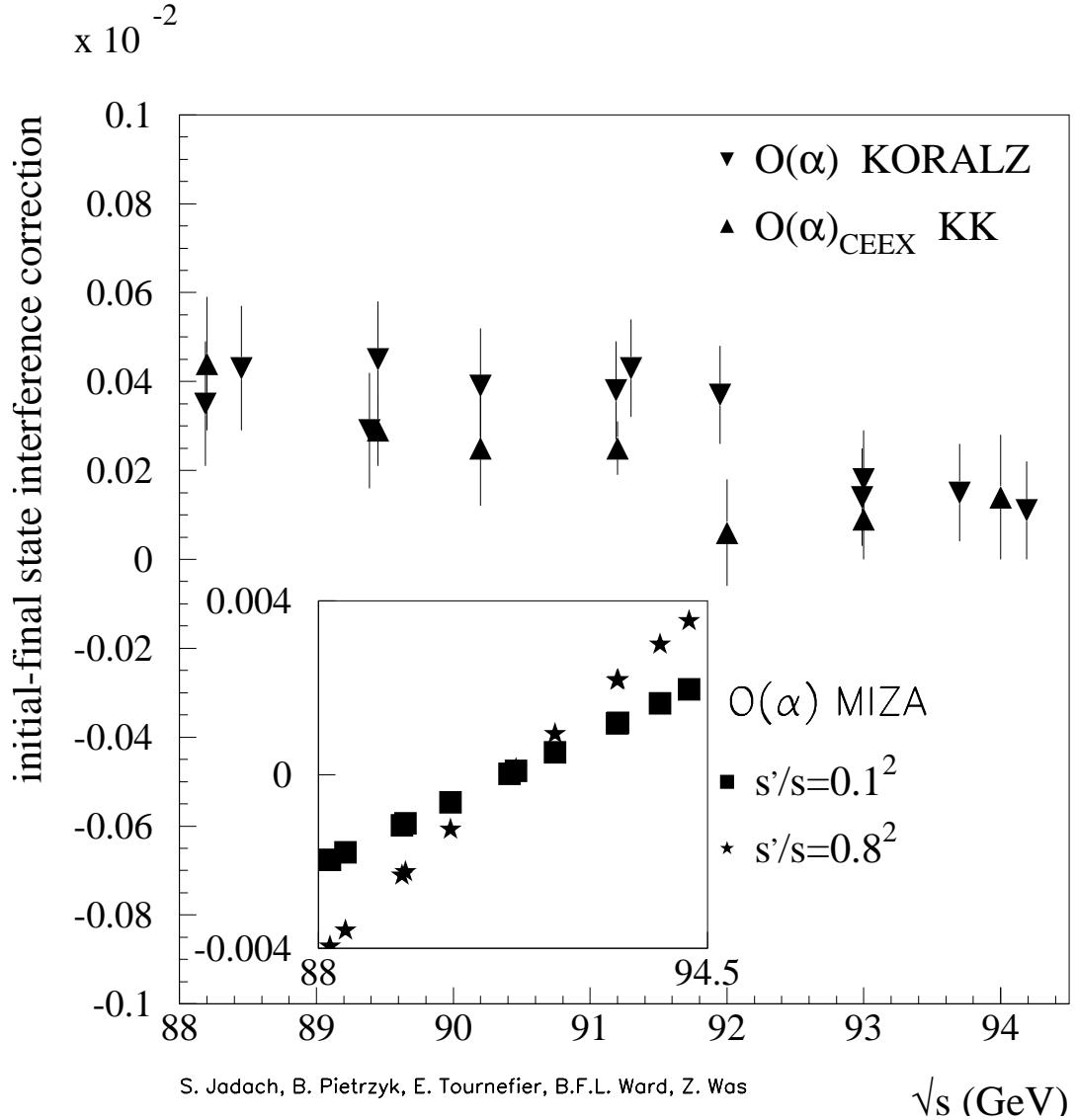


Figure 2: Corrections to the ALEPH experimental efficiency due to implementation of the Initial–Final-State Interference (IFI) calculated with the  $\mathcal{KK}$  and a modified version of the KORALZ MC programs. The inset shows the dependence of the IFI corrections on the  $s'$  cut. Note that both the vertical and horizontal scales of the figure are different from those of the inset.

properties of IFI before we go into details. The important properties of IFI corrections at and around the  $Z$  peak, as seen explicitly in the  $\mathcal{O}(\alpha^1)$  analytical results, are that (a) the IFI correction is suppressed by a factor  $\Gamma_Z/M_Z$  (or  $(s - M_Z^2)/M_Z^2$ ), (b) for  $e^-e^+ \rightarrow f\bar{f}$  it

Test option	Shift of $M_Z$
No IFI	+0.04 MeV
Only lepton IFI	-0.17 MeV
Only hadron IFI	+0.17 MeV
Lepton and 50% hadron IFI	-0.07 MeV

Table 1: Change of the  $M_Z$  on the line-shape fit output with different implementations of the Initial–Final-State Interference (IFI) correction with respect to the full calculations including IFI correction.

does not contain mass logarithms  $\ln(s/m_e^2)$  or  $\ln(s/m_f^2)$ . It is of order

$$\frac{\sigma^{IFI}}{\sigma} \sim Q_e Q_f \frac{\alpha}{\pi} \text{Max} \left\{ \frac{\Gamma_Z}{M_Z} ; \frac{s - M_Z^2}{M_Z^2} \right\}$$

where  $Q_e$  and  $Q_f$  are electric charges of the electron and final-state fermion. The above correction is therefore numerically small, provided cuts on photon energies are absent or loose. The reason for the  $\Gamma_Z/M_Z$  suppression is the time separation between the production and decay processes of the relatively long-lived  $Z$ , while the reason for the absence of mass logarithms is that the IFI interference comes from the angular range where the photon is at roughly equal angular distance from both initial- and final-state fermions<sup>3</sup>. The above two reasons are very elementary and simple; they are not limited to any perturbative order: they will lead to the same suppression pattern at higher orders. If in the actual calculation they do not, then it is only because of some bad unphysical and/or technical approximation made in the perturbative calculations, which will be cured in a more complete calculation. For instance, the overall size of the interference correction is the result of delicate cancellation of a rather big infrared (IR) infinite contribution from real and virtual QED corrections. If these cancellations are disturbed, the higher orders may get artificially enhanced. Typically, if the  $\mathcal{O}(\alpha^1)$  IFI corrections, in an attempt at including big non-interference higher orders, are improperly “folded in” with the big non-interference higher-order ones, then we may get an unphysical enhancement of the IFI corrections. Of course, we expect some interplay of the IFI with big non-interference corrections. However, this should generally happen in a “multiplicative way”, such that the real-virtual cancellations for the interference contributions are maximally preserved.

One example of improper technical approximation enhancing IFI corrections is already known within  $\mathcal{O}(\alpha^1)$ ; in the calculation of Ref. [10], the authors have applied the so-called “pole approximation”, generally accepted and known to work well, for the box diagram and soft bremsstrahlung. This damages the delicate cancellation between the two, resulting in a falsely increased interference correction around the  $Z$  peak (but not at the peak itself). This approximation does not really violate the  $\Gamma/M_Z$  suppression of IFI. It only causes the coefficient in front of  $(s - M_Z^2)/M_Z^2$  to be incorrect.

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<sup>3</sup> This can be clearly seen for real photons, and the same has to be true for the virtual ones (IR cancellations).

What is the generic size of the IFI at  $\mathcal{O}(\alpha^2)$  near the  $Z$ ? We expect the contribution to be of order:

$$\frac{\sigma^{IFI}}{\sigma} \sim \text{Max}\left\{\frac{\Gamma_Z}{M_Z}; \frac{s - M_Z^2}{M_Z^2}\right\} Q_e Q_f \left(\frac{\alpha}{\pi}\right) \times Q_e^2 \left(\frac{\alpha}{\pi}\right) \ln \frac{s}{m_e^2} \ln \frac{\Gamma_Z}{E_{beam}}, \quad (1)$$

where the  $(\alpha/\pi) \ln(s/m_e^2) \ln(\Gamma_Z/E_{beam})$  part is the Sudakov double logarithm from the non-interference ISR, essentially the same which reduces by 30% the  $Z$  peak cross section at  $\mathcal{O}(\alpha^1)$ . The  $\ln(\Gamma_Z/E_{beam}) \sim \ln(E_{max}/E_{beam})$  is due to the cut on ISR photon energy induced by a resonance behaviour in the Born cross section.

In Fig. 1 we show results for the IFI contribution to the total muonic and hadronic cross section from the  $\mathcal{O}(\alpha^1)$  calculations of MIZA, ZFITTER,  $\mathcal{O}(\alpha^1)$  KORALZ and from the  $\mathcal{KK}$  MC [18], which is the only available calculation beyond  $\mathcal{O}(\alpha^1)$  (see below for more details). As we see, all four  $\mathcal{O}(\alpha^1)$  results agree fairly well, as they should (only a technical problem could cause a difference) while results from the exponentiated *Coherent Exclusive Exponentiation* (CEEX)  $\mathcal{O}(\alpha^1)_{CEEX}$ , calculation of  $\mathcal{KK}$  MC differs from previous one in the slope of the energy dependence by about 25%<sup>4</sup>. This is exactly the size we expect from eq. (1). Note that for leptons and  $\mathcal{O}(\alpha^1)_{CEEX}$  the IFI correction has its zero at a higher value of  $\sqrt{s}$  than for  $\mathcal{O}(\alpha^1)$ . For hadrons the above effect is less pronounced, because of cancellations between different flavours.

Let us now briefly explain why exponentiation is the correct and economical solution for getting a hint on IFI beyond  $\mathcal{O}(\alpha^1)$ . Already from the  $\mathcal{O}(\alpha^1)$  we learn one important lesson: the IFI corrections are large if we apply a strong cut on photon energies. This is the only possible big enhancement factor at this order. In fact these corrections may be so large that it is necessary to sum them up to infinite order, i.e. to exponentiate. This is precisely what is done in the  $\mathcal{KK}$  MC, with the *Coherent Exclusive Exponentiation* (CEEX) described in Ref. [19], employing spin amplitudes of Ref. [20]<sup>5</sup>. Why is exponentiation the correct approach to IFI beyond  $\mathcal{O}(\alpha^1)$ ? First of all, because exponentiation is basically “multiplicative” and it will thus preserve at higher orders the virtual–real cancellations that are at the heart of the IFI suppression. Secondly, it sums up the only possible enhancement factor in IFI, which is due to the strong cut ( $\sim \Gamma_Z$ ) on photon energy.

In practice one has to be careful. For instance, also the important virtual correction [22]  $Q_e Q_f (\alpha/\pi) \ln(t/u) \ln((M_Z - iM_Z \Gamma_Z - s)/M_Z)$  has to be summed up to infinite order (included in the exponential form factor). As discussed in Ref. [19], failure to do this leads to a disturbance of the virtual–real cancellation and consequently to lack of the suppression factor ( $\Gamma_Z/M_Z$ ) at  $\mathcal{O}(\alpha^2)$ , resulting in a dramatic unphysical increase of the IFI correction close to the  $Z$  peak.

Let us finally note that also the  $\mathcal{KK}$  MC [18] provides interesting results on IFI for LEP2 energies. They will be presented in a separate publication [23].

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<sup>4</sup> Different conventions appear in the literature on how the *relative* interference correction should be presented. In our case we calculate the correction always within the given order of perturbation expansion. Thus, e.g.  $\delta_{int}(\mathcal{O}(\alpha^1)) = [\sigma_{int}(\mathcal{O}(\alpha^1)) - \sigma_{no\ int}(\mathcal{O}(\alpha^1))]/\sigma_{no\ int}(\mathcal{O}(\alpha^1))$  and  $\delta_{int}(\mathcal{O}(\alpha^1)_{CEEX}) = [\sigma_{int}(\mathcal{O}(\alpha^1)_{CEEX}) - \sigma_{no\ int}(\mathcal{O}(\alpha^1)_{CEEX})]/\sigma_{no\ int}(\mathcal{O}(\alpha^1)_{CEEX})$ .

<sup>5</sup> In the first multiphoton MC with exponentiation beyond  $\mathcal{O}(\alpha^1)$  of Ref. [21], which is used in the multiphoton KORALZ, the IFI is completely omitted.

Another source of the theoretical systematics is related to QCD corrections for the hadronic Z decays, as well as vacuum polarization. The above discussion of IFI is made in the framework of pure QED. However, quarks may emit final-state gluons as they emit initial- and/or final-state photons. The correction of the lowest possible order related to the above configuration, taking only the QED IFI part, is expected to be

$$\frac{\delta\sigma^{IFI}}{\sigma} \sim \frac{\Gamma_Z}{M_Z} Q_e Q_q \frac{\alpha}{\pi} \times \frac{\alpha_s}{\pi} \sim 4 \times 10^{-6}$$

provided we do not apply very sharp acceptance cuts on photon and gluon energies (or do not try to look into a sub-sample with isolated photon or hard gluon). This correction cannot be enhanced by the logarithms of the electron mass  $|\ln(m_e^2/s)| \sim 20$ . This is because the IFI contribution can only arise from the phase-space region where the photon angle from initial- and final-state fermions is large and about the same, while the mass logarithms  $\ln(m_e^2/s)$  and  $\ln(m_f^2/s)$  come mostly from the small photon angles. This mechanism cannot be changed by hadronization of additional gluon emission because hadrons in jets are narrowly correlated and a jet is acting coherently as an effective object of almost zero or low electric charge. None of the perturbative or non-perturbative self-interaction within a jet can change this simple fact. Nor can the gluon emission induce a logarithm of the quark mass, even in the presence of an additional photon, because of the Kinoshita-Lee-Nauenberg theorem.

We shall assume here that these QED–QCD corrections can change the  $\mathcal{O}(\alpha^1)$  IFI correction for hadronic final states by the rather conservatively large amount of 50%. The Z mass on the MIZA fit output is then changing by less than 0.1 MeV, when the hadronic IFI correction is changed by 50% as seen in Table 1. In this way the theoretical error due to the possible  $\mathcal{O}(\alpha_s \alpha_{QED})$  correction largely dominates the other possible theoretical uncertainties discussed above. Therefore the error of 0.1 MeV on the  $M_Z$  is the total conservative theoretical uncertainty on the IFI calculations.

	$\sigma_{had}^0$	$M_Z$	$\Gamma_Z$
ISR	$1 \times 10^{-4}$	Negl.	Negl.
Pairs	$1.8 \times 10^{-4}$	0.1 MeV	0.1 MeV
IFI	negl.	0.1 MeV	negl.
total	$2 \times 10^{-4}$	0.15 MeV	0.1 MeV

Table 2: Theoretical uncertainties in the MIZA line-shape fit results coming from the precision of the calculation of Initial State Radiation ISR [4], fermion pair production [4] and Initial–Final-State Interference IFI.

Theoretical uncertainties on the MIZA line-shape fit results coming from the precision of the calculation of ISR [4], fermion-pair production [4] and IFI are given in Table 2.

The precision of the IFI contribution is discussed above, including Fig. 1 for the low values of the  $s'/s$  cut used for the published values of the cross-section measured by LEP.

These low values are similar to the ones used for the hadronic event selection by the LEP experiments, but they are much lower than the ones used typically for the leptonic selection. A lepton acollinearity cut of  $20^\circ$  is made by the ALEPH Collaboration [14], corresponding approximately to  $\sqrt{s'} > 0.8\sqrt{s}$ . Calculation of the experimental efficiency using a program without implementation of the IFI contribution could lead to an important experimental bias, as seen in the inset in Fig. 2 where  $\mathcal{O}(\alpha^1)$  calculations in MIZA [6] (using formulas from Ref. [11]) are compared for two different values of the  $s'/s$  cut. The corrections for the experimental efficiency could be as high as the difference between these two distributions in the case of a full geometrical acceptance of the detector. The real corrections are, however, smaller, owing to angular cuts. For example, in the ALEPH leptonic selection it is required that  $|\cos \Theta^*| < 0.9$ , where  $\Theta^*$  is the centre of mass scattering angle. The IFI is definitely bigger at high  $|\cos \Theta^*|$ . Figure 2 shows the corrections to the ALEPH muon efficiency calculated using the KK MC program. This correction is equal to the relative difference of the acceptance calculated with and without IFI. The results of the  $\mathcal{O}(\alpha^1)$  calculations using a modified version of the KORALZ MC program are similar. The necessary modifications, which are absent in the public version of the code, consist of the use of weighted events allowing the shift of the so-called  $k_0$  cut-off to an arbitrary low value. This enables one to remove the related bias; see refs. [13, 12] and references therein for technical details. The  $k_0$  cut-off separates the region of the phase space where the kinematics of the event includes the hard photon 4-momentum from the one where it is integrated and summed with virtual corrections.

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